Roll No. ....

E–3827

M. A./M. Sc. (Final) EXAMINATION, 2021

MATHEMATICS

(Compulsory)

Paper Second

(Partial Differential Equations and Mechanics)

Time : Three Hours] [*Maximum Marks : 100*

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove the mean-value formula for Laplace's equation.

(b) Prove the Kirchhoff's formula for the wave equation.

(c) Prove that there exists for each pair of integers $k, l = 0, 1, \ldots\ldots\ldots\ldots$ a constant $C_{k,l}$ such that :

$$\max_{C(x,t;\frac{r}{2})} |D_x^k D_t^l u| \leq \frac{C_{kl}}{r^{k+2l+n+2}} \| u \|_{\alpha'(C(x,t,r))}$$

for all closed circular cylinders

$$C(x,t;\frac{r}{2}) \subset C(x,t;r) \subset U_T$$

and all solutions u of the heat equation in U_T.

Unit—II

2. (a) Using separation of variables, solve the initial value problem for the heat equation :

$$u_t - \Delta u = 0 \text{ in } U \times (0,\infty)$$

$$u = 0 \text{ on } \partial U \times [0,\infty)$$

$$u = g \text{ on } U \times \{t = 0\},$$

where $U \subset R^n$ be a bounded, open set with smooth boundary and $g : U \to R$ is given.

(b) Write short notes on the following :

(i) Geometric optics

(ii) Homogenization

(c) State and prove Euler-Lagrange equations.

Unit—III

3. (a) State and prove Donkin's theorem.

(b) Explain virtual work and derive the D' Alembert's principle.

(c) Find the curve joining two given points, which generates the surface of the minimum area when rotated about the x-axis.

Unit—IV

4. (a) Derive Whittaker's equations.

(b) Prove the relation between the Lagrange and Poisson brackets.

(c) Derive Hamilton's canonical equations using Hamilton's principle.

Unit—V

5. (a) Derive Poisson's equation (Cartesian) from Gauss' theorem.

(b) Find the attraction of a thin uniform rod AB at an external point P.

(c) Show that the work done in collecting the particles of a thin circular disc attracting according to Newtonian law from an infinite distance is $\frac{8}{3} \mathrm{Y} \frac{\mathrm{M}^2}{\pi a}$, where 'M' is the mass of the disc and 'a' is its radius.